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AZIMUTHAL CORRELATIONS IN PHOTON-PHOTON COLLISIONS

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Abstract

Using the general helicity formula for $\gamma^*\gamma^*$ collisions, we are showing that it should be possible to determine a number of independent “structure functions”, i.e. linear combinations of elements of the two-photon helicity tensor, through azimuthal correlations in two-body or quasi two-body reactions induced by the photon-photon interaction, provided certain experimental conditions are satisfied. Numerical results of our computations are presented for some particular processes and dynamic models.

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I. INTRODUCTION

Azimuthal correlations in photon-photon collisions have been studied, in the past, in a number of papers where, in particular, the single-tag configuration was considered [1–6]. One paper was also devoted to the study of those correlations in a double-tag configuration with both electrons being tagged at small angle [7]. Let us mention, in addition, a paper [8] where the authors investigated azimuthal correlations in pair production in a no-tag configuration, using the acoplanarity of the produced particles in the lab frame.

The purpose of the present paper is to show how the potential of azimuthal correlations, as they can be derived from the general helicity formula for $\gamma^*\gamma^*$ collisions in the case of 2-body or quasi 2-body reactions, can be exploited, in various experimental configurations, in order to extract a maximum of physical information from measurements of those reactions. For each of those configurations, we define the corresponding experimental constraints to be applied.

In section II we write down the general helicity formula as a sum of 13 terms involving each a different dependence on azimuthal angles. In section III we apply that formula to four particular experimental configurations where the analysis of azimuthal correlations should allow one to determine a number of structure functions F_i . Our treatment of the first two is rather trivial ; that of the third and fourth one is more sophisticated, since it involves the use of “azimuthal selection”, as we shall define it. Since the fourth configuration appears to be the most promising one and has not been considered elsewhere, we compute, in section IV, a number of corresponding applications to particular processes and dynamic models. Section V contains a brief discussion and conclusion. Kinematic constraints to be applied in configurations 2 and 4 are being computed in an Appendix.

II. THE GENERAL HELICITY FORMULA FOR $\gamma^*\gamma^*$ COLLISIONS

The general helicity formula for two-body or quasi two-body reactions induced by the photon-photon interaction, i.e. for processes of the type $e e' \rightarrow e e' a b$ (where b may be a system of particles instead of a single one), as shown by Fig. 1, has been in the literature for a long time [9–14]. While at the start this formula contains $3^4 = 81$ terms, since the helicity matrix of either photon is composed of 3×3 elements, that number is considerably reduced by applying first principles, namely hermiticity, parity conservation and rotational invariance. Gathering together terms which have the same behaviour with respect to azimuthal angles, we get a 13-term formula :

$$\frac{sQ^2Q'^2}{e^8} d\sigma = \frac{1}{(1-\epsilon)(1-\epsilon')} F d\text{Lips} \quad (1)$$

with :

$$\begin{aligned} F = & F_1 - 2\sqrt{\epsilon(1+\epsilon)}F_2 \cos \varphi_a - 2\epsilon F_3 \cos 2\varphi_a \\ & + 2\sqrt{\epsilon'(1+\epsilon')}F_4 \cos(\varphi - \varphi_a) - 2\epsilon' F_5 \cos 2(\varphi - \varphi_a) \\ & - 2\sqrt{\epsilon(1+\epsilon)\epsilon'(1+\epsilon')}F_6 \cos \varphi + \epsilon\epsilon' F_7 \cos 2\varphi \\ & - 2\sqrt{\epsilon(1+\epsilon)\epsilon'(1+\epsilon')}F_8 \cos(2\varphi_a - \varphi) + \epsilon\epsilon' F_9 \cos 2(2\varphi_a - \varphi) \\ & - 2\epsilon\sqrt{\epsilon'(1+\epsilon')}F_{10} \cos(\varphi + \varphi_a) + 2\epsilon'\sqrt{\epsilon(1+\epsilon)}F_{11} \cos(2\varphi - \varphi_a) \\ & - 2\epsilon\sqrt{\epsilon'(1+\epsilon')}F_{12} \cos(3\varphi_a - \varphi) + 2\epsilon'\sqrt{\epsilon(1+\epsilon)}F_{13} \cos(2\varphi - 3\varphi_a) \end{aligned} \quad (2)$$

Here the quantities F_i ($i = 1...13$) are linear combinations of elements of the helicity tensor associated with the process $\gamma\gamma' \rightarrow a b$; these quantities, which are typical structure functions (F_1 is the diagonal structure function, while all others may be called “interference structure functions”), are given as follows :

$$\begin{aligned} F_1 &= F_{++,++} + F_{+,-,-} + 2\epsilon F_{00,++} + 2\epsilon' F_{++,00} + 2\epsilon\epsilon' F_{00,00} ; \\ F_2 &= \Re e(F_{+0,++} - F_{0-,++} + 2\epsilon' F_{+0,00}) ; & F_3 &= \Re e F_{+-,++} + \epsilon' F_{+-,00} ; \\ F_4 &= \Re e(F_{++,+0} - F_{++,0-} + 2\epsilon F_{00,+0}) ; & F_5 &= \Re e F_{++,+-} + \epsilon F_{00,+-} ; \end{aligned}$$

$$\begin{aligned}
F_6 &= \Re e(F_{+0,+0} - F_{+0,0-}); & F_7 &= F_{+-,+-}; & F_8 &= \Re e(F_{+0,0+} - F_{+0,0-}); \\
F_9 &= F_{+-,-+}; & F_{10} &= \Re e F_{+-,+0}; & F_{11} &= \Re e F_{+0,+-}; \\
F_{12} &= \Re e F_{+-,0+}; & F_{13} &= \Re e F_{0+,+-};
\end{aligned} \tag{3}$$

where the tensor elements $F_{m\bar{m},n\bar{n}}$ are defined as

$$F_{m\bar{m},n\bar{n}} = \sum \mathcal{M}_{mn}^{(\gamma\gamma' \rightarrow ab)} \mathcal{M}_{\bar{m}\bar{n}}^{*(\gamma\gamma' \rightarrow ab)} \tag{4}$$

calling $m(\bar{m})$ resp. $n(\bar{n})$ the helicities of γ resp. γ' ; the symbol \sum indicates summation over the spin states of a and b . The elements $F_{m\bar{m},n\bar{n}}$ depend on W^2, Q^2, Q'^2 and χ , defining : $W^2 = (q + q')^2$, $Q^2 = -q^2$, $Q'^2 = -q'^2$ (where q and q' are the respective four-momenta of γ and γ'), and calling χ the polar angle of a with respect to γ in the $\gamma\gamma'$ center-of-mass frame. Let us remark that hermiticity entails the relation $F_{m\bar{m},n\bar{n}} = F_{\bar{m}m,\bar{n}n}^*$, while parity conservation and rotational invariance entail $F_{m\bar{m},n\bar{n}} = (-1)^{m+\bar{m}+n+\bar{n}} F_{-m-\bar{m},-n-\bar{n}}$.

In addition we use the following notations : φ is the azimuthal angle of e' with respect to e in the $\gamma\gamma'$ c. m. frame with the z axis oriented along the three-momentum of the photon γ ; φ_a is the azimuthal angle of a with respect to e in the same frame. The polarization parameters ϵ and ϵ' of, respectively, the photons γ and γ' are given as follows :

$$\begin{aligned}
\epsilon &= \frac{2x's(x's - W^2 - Q^2 - Q'^2) + 2Q^2Q'^2}{(x's - W^2 - Q^2 - Q'^2)^2 + x'^2s^2 - 2Q^2Q'^2} \\
\epsilon' &= \frac{2xs(xs - W^2 - Q^2 - Q'^2) + 2Q^2Q'^2}{(xs - W^2 - Q^2 - Q'^2)^2 + x^2s^2 - 2Q^2Q'^2}
\end{aligned} \tag{5}$$

where s is the total energy squared in the overall center-of-mass frame, while x and x' are defined as: $x = (q \cdot p'_0)/(p_0 \cdot p'_0)$, $x' = (p_0 \cdot q')/(p_0 \cdot p'_0)$, calling p_0, p'_0 the respective four-momenta of e_0, e'_0 . Notice that in (5) we have neglected the electron mass ; actually one has $\epsilon(\epsilon') \rightarrow 0$ when $Q^2(Q'^2)$ reaches its minimal value.

The Lorentz-invariant phase space is given by

$$d\text{Lips} = \frac{1}{(2\pi)^{3n_f-4}} \delta^4 \left(\sum_i p_i - \sum_f p_f \right) \prod_f \frac{d^3 \mathbf{p}_f}{p_f^0} \tag{6}$$

where n_f is the number of final particles, and p_i, p_f are the respective four-momenta of initial and final particles, while the superscript 0 indicates the energy component. (If b

is a system of particles, an integration over all corresponding 3-momenta, except one, is implicitly included.)

Finally W^2 is given by

$$W^2 = -Q^2 - Q'^2 + \frac{Q^2 Q'^2}{s} + xx's - 2QQ'\sqrt{(1-x)(1-x')}\cos\varphi^{\text{lab}} \quad (7)$$

where φ^{lab} is the azimuthal angle of e' with respect to e in the lab frame with the z axis oriented along the three-momentum of the incident electron e_0 .

III. PARTICULAR CONFIGURATIONS

A. Configuration 1 :

Double-tag measurement, extrapolating the central-detector acceptance to 4π

We here assume a measurement where both outgoing electrons are tagged, and a so-called “unfolding” procedure is used in order to extrapolate the acceptance of the central detector to 4π . This allows one to integrate formula (2) over φ_a between 0 and 2π , so that one obtains the three-term formula (see Ref. [1]) :

$$(2\pi)^{-1} \int F d\varphi_a = F_1 - 2\sqrt{\epsilon(1+\epsilon)\epsilon'(1+\epsilon')} F_6 \cos\varphi + \epsilon\epsilon' F_7 \cos 2\varphi \quad (8)$$

Actually formula (8) can hardly be exploited in that form for an azimuthal-correlation study, for the following reason : Since F_6 involves longitudinal (helicity 0) components of both photons, it can be shown by general arguments (see section III of Ref. [14]) to stay non-negligible only when both outgoing electrons are tagged at large angles (i.e. $Q, Q' \approx W$) [15]. But in that case W depends on φ (as can be inferred from formula (7), since φ^{lab} is correlated with φ), and therefore the functions F_i , as well as their coefficients, also depend on φ . It is then obvious that it would become much more complicated to extract the structure functions from the analysis of the φ distribution (all the more as the integrated cross sections are expected to be very small in that kinematic situation).

One is thus led to consider the case where at least one of the outgoing electrons (for instance e') is tagged at small angle (i.e. $Q' \ll W/2$). We now use formula (7) and in addition the relation

$$\cos \varphi = \cos \varphi^{\text{lab}} + \frac{QQ'}{W^2 + Q^2} \frac{2 - x - x'}{\sqrt{(1-x)(1-x')}} \sin^2 \varphi^{\text{lab}} + O\left(\frac{Q'^2}{W^2}\right) \quad (9)$$

That relation shows that - except for a small range near $\varphi^{\text{lab}} = \pi/2$, and for marginal ranges of x and x' that one can suppress by setting upper limits on those variables - one is allowed to identify φ with φ^{lab} . We thus get :

$$W^2 = -Q^2 + xx's - 2QQ' \sqrt{(1-x)(1-x')} \cos \varphi + O(Q'^2) \quad (10)$$

Still one may consider two different experimental situations : (A) both electrons are tagged at small angle ($Q, Q' \ll W/2$) ; (B) e' is tagged at small angle, e at large angle ($Q' \ll W/2, Q \lesssim W$). In both cases W tends to become independent of φ , i.e. :

$$(A) \ W^2 \simeq xx's \quad \text{resp.} \quad (B) \ W^2 \simeq -Q^2 + xx's, \quad (11)$$

while at the same time the condition for neglect of longitudinal contributions (Ref. [14], formula (3.5)), i.e. $Q' \ll (W^2 + Q^2 - Q'^2)/(2W)$, is satisfied, so that F_6 goes to zero. One is then left with the two-term formula

$$(2\pi)^{-1} \int F d\varphi_a = F_1 + \epsilon \epsilon' F_7 \cos 2\varphi \quad (12)$$

where in case (A) one has

$$F_1 = F_{++,++} + F_{++,--} ; \quad F_7 = F_{+-,+-} ; \quad (13)$$

with the helicity-tensor elements depending only on W^2 and χ , while in case (B) one gets

$$F_1 = F_{++,++} + F_{++,--} + 2\epsilon F_{00,++} ; \quad F_7 = F_{+-,+-} ; \quad (14)$$

with the helicity-tensor elements depending on W^2 , Q^2 and χ .

F_1 , being composed only of diagonal elements of the helicity-tensor, is to be identified, apart from kinematic factors, with the differential cross section $d\sigma_{\gamma\gamma}/d(\cos \chi)$ in case (A),

resp. $(d\sigma_T + \epsilon d\sigma_L)/d(\cos \chi)$ in case (B), for the reaction $\gamma\gamma' \rightarrow ab$; for instance, in the case where (a, b) is a pair of charged particles, one gets

$$\frac{d\sigma_{\gamma\gamma}}{d(\cos \chi)} \quad \text{resp} \quad \frac{d\sigma_T + \epsilon d\sigma_L}{d(\cos \chi)} = \frac{\Lambda^{1/2}(W^2, m_a^2, m_b^2)}{64\pi W^4} F_1 \quad (15)$$

where Λ is defined as : $\Lambda(A, B, C) = A^2 + B^2 + C^2 - 2AB - 2AC - 2BC$. That cross section can obviously be determined as well in the no-tag resp. the single-tag mode. Double tagging, with the constraints defined in order to allow for an azimuthal-correlation analysis, provides the possibility of extracting an additional structure function (F_7) ; the price to pay is, of course, a lowering of the yield obtained.

Notice that, at $Q' \ll W/2$, formula (5) becomes considerably simplified, as one gets

$$\epsilon = \frac{1-x}{1-x+x^2/2}; \quad \epsilon' = \frac{1-x'}{1-x'+x'^2/2} \quad (16)$$

Finally let us remark that, since $\varphi \simeq \varphi^{\text{lab}}$, as shown by formula (9), and since one may assume the electron-tagging systems to be cylindrically symmetric, there should be no distorsion, due to the apparatus, of the φ distribution.

B. Configuration 2 :

Single-tag measurement

In that configuration the untagged (or antitagged) electron, e.g. e' , is predominantly emitted very close to 0° , so that Q' becomes essentially negligible as compared with $W/2$, which has the obvious consequences that : (i) W becomes independent of φ (see again formula (10)) ; (ii) $\varphi \simeq \varphi^{\text{lab}}$ (formula (9)) and therefore, assuming here again the electron tagging system to be cylindrically symmetric, there should be no distorsion of the φ distribution due to the apparatus. We may thus integrate formula (2) over φ between 0 and 2π , so that we obtain the three-term formula :

$$(2\pi)^{-1} \int F d\varphi = F_1 - 2\sqrt{\epsilon(1+\epsilon)}F_2 \cos \varphi_a - 2\epsilon F_3 \cos 2\varphi_a \quad (17)$$

with

$$\begin{aligned}
F_1 &= F_{++,++} + F_{++,--} + 2\epsilon F_{00,++}; & F_2 &= \Re e(F_{+0,++} - F_{0-,++}); \\
F_3 &= \Re e F_{+-,++}.
\end{aligned} \tag{18}$$

Again ϵ, ϵ' are given by formula (16). In Ref. [6] formula (18) has been applied to muon and pion pair production, using two different models for the latter. However, in that paper, the problem of experimental constraints to be applied in order to suppress the distortion of the φ_a distribution, induced by the limited acceptance of the central detector, was left aside. Actually one should here assume that the latter has an “almost 4π ” acceptance. Yet a kinematic study shows that, even if the acceptance cuts of the central detector remain very small, the cuts induced by them in the polar emission angle of the particles produced in the $\gamma\gamma'$ c. m. frame may be large, not confined to the margins of phase space, and azimuth-dependent (as was already noticed in Ref. [5]). Therefore additional constraints should be imposed in order to minimize the latter cuts. This problem, which appears as well in configuration 4, is treated in the Appendix of this paper.

Notice that in the single-tag case W can only be determined by measuring all particles produced. When this is not possible (in the case of multi-particle final-states), the fact that $W_{\text{vis}} \neq W$ is an additional source of complications.

Let us mention that a measurement of this type, involving muon pair production, has recently been performed by the L3 Collaboration at LEP (CERN) [16].

C. Configuration 3 :

Double-tag measurement at small angles

We here assume that the electron-tagging angles are small enough to ensure that $Q, Q' \ll W/2$, so that in formula (2) we may neglect all longitudinal helicity-tensor elements, i.e. those with at least one 0 subscript. We are thus left with the five-term formula (see Ref. [12], Eq. (5.33)) :

$$\begin{aligned}
F &= F_1 - 2\epsilon F_3 \cos 2\varphi_a - 2\epsilon' F_5 \cos 2(\varphi - \varphi_a) \\
&\quad + \epsilon\epsilon' F_7 \cos 2\varphi + \epsilon\epsilon' F_9 \cos 2(2\varphi_a - \varphi)
\end{aligned} \tag{19}$$

with

$$\begin{aligned}
F_1 &= F_{++,++} + F_{++,--}; & F_3 &= \Re e F_{+-,++}; \\
F_5 &= \Re e F_{++,+-}; & F_7 &= F_{+-,+-}; & F_9 &= F_{+-,-+};
\end{aligned} \tag{20}$$

where the helicity-tensor elements depend on W^2 and χ ; ϵ, ϵ' are again given by formula (16). At this point we note that, if the whole range $0 < \varphi < 2\pi, 0 < \varphi_a < 2\pi$ is available, one may use the orthogonality of the functions $\{1, \cos 2\varphi_a, \cos 2(\varphi - \varphi_a), \cos 2\varphi, \cos 2(2\varphi_a - \varphi)\}$ in that range and thus extract F_1, \dots, F_5 by projecting formula (19) on those functions. However it seems preferable, in order to get more information, to apply what we call “azimuthal selection”, i.e. to select individual distributions with respect to the azimuthal angles involved, as follows : let us assume, for instance, that we are interested in the distribution with respect to the azimuthal angle $\hat{\varphi} = 2\varphi_a - \varphi$. Switching from our system of azimuthal variables φ, φ_a to the system $\varphi, \hat{\varphi}$, we write :

$$\begin{aligned}
F &= F_1 - 2\epsilon F_3 \cos(\varphi + \hat{\varphi}) - 2\epsilon' F_5 \cos(\varphi - \hat{\varphi}) \\
&\quad + \epsilon\epsilon' F_7 \cos 2\varphi + \epsilon\epsilon' F_9 \cos 2\hat{\varphi}
\end{aligned} \tag{21}$$

Integrating over φ between 0 and 2π , we then get :

$$(2\pi)^{-1} \int F d\varphi = F_1 + \epsilon\epsilon' F_9 \cos 2\hat{\varphi} \tag{22}$$

Similarly we can select three other azimuthal-angle distributions :

(using the system of variables φ, φ_a) :

$$(2\pi)^{-1} \int F d\varphi = F_1 - 2\epsilon F_3 \cos 2\varphi_a \tag{23}$$

(using the system of variables $\varphi, \varphi'_a = \varphi - \varphi_a$) :

$$(2\pi)^{-1} \int F d\varphi = F_1 - 2\epsilon' F_5 \cos 2\varphi'_a \tag{24}$$

(using the system of variables φ, φ_a) :

$$(2\pi)^{-1} \int F d\varphi_a = F_1 + \epsilon\epsilon' F_7 \cos 2\varphi \tag{25}$$

Here we have again made use of the fact that the φ -dependence of W can be neglected according to formula (10). In addition we have implicitly assumed that there is no distortion of the φ distribution due to the apparatus (i.e. φ varies between 0 and 2π whatever the values of the other variables within the phase space considered) ; that assumption is justified, here again, by the fact that $\varphi \simeq \varphi^{\text{lab}}$ (formula (9)), and that we may suppose both electrons to be tagged in a cylindrically symmetric way.

Finally we have implicitly assumed that there is no distortion of the φ_a distribution. This assumption, as already discussed in Ref. [7], requires a somewhat more stringent condition to be imposed on Q and Q' , namely : $Q, Q' \ll (W/2) \sin \chi$; that means that the transverse momenta of both photons in the lab frame can be neglected with respect to the transverse momentum of a in the $\gamma\gamma'$ c. m. frame. Under that condition the $\gamma\gamma'$ collision axis tends to be identical with the colliding-beam axis ; consequently one gets $\varphi_a \simeq \varphi_a^{\text{lab}}$, and it thus becomes sufficient to assume that the central detector is, as well, cylindrically symmetric. The condition here defined implies of course that an appropriate lower limit is assigned to $\sin \chi$.

As one sees from formulas (21)-(24), azimuthal selection should allow one, in the configuration considered, to extract four azimuthal-angle distributions, and correspondingly five structure functions, from the data obtained in a single measurement. It must however be remarked that because of left-right symmetry the φ_a and φ'_a distributions, and correspondingly the structure functions F_3 and F_5 , should be identical. Thus, we get in fact three independent azimuthal correlations, allowing for the determination of four independent structure functions.

In Ref. [7] formulas (22)-(25) have been applied to lepton and pion pair production.

It is to be mentioned that an experiment of that type is presently being prepared at the low-energy electron-positron collider DAΦNE at Frascati [17].

D. Configuration 4 :

Double-tag measurement with one electron tagged at large angle and the other one at small angle

As in case (B) of configuration 1, we here assume that the electron e is tagged at large angle ($Q \lesssim W$), while e' is assumed to be tagged at small angle ($Q' \ll W/2$). Thus W becomes practically independent of φ , according to formula (10), while at the same time all helicity-tensor elements with at least one 0 helicity subscript for the right-hand photon tend to vanish. Formula (2) is then reduced to an eight-term formula :

$$\begin{aligned}
F = & F_1 - 2\sqrt{\epsilon(1+\epsilon)}F_2 \cos \varphi_a - 2\epsilon F_3 \cos 2\varphi_a \\
& - 2\epsilon' F_5 \cos 2(\varphi - \varphi_a) + \epsilon\epsilon' F_7 \cos 2\varphi + \epsilon\epsilon' F_9 \cos 2(2\varphi_a - \varphi) \\
& + 2\epsilon'\sqrt{\epsilon(1+\epsilon)}F_{11} \cos(2\varphi - \varphi_a) + 2\epsilon'\sqrt{\epsilon(1+\epsilon)}F_{13} \cos(2\varphi - 3\varphi_a)
\end{aligned} \tag{26}$$

with

$$\begin{aligned}
F_1 &= F_{++;++} + F_{++;-} + 2\epsilon F_{00,++}; & F_2 &= \Re e(F_{+0,++} - F_{-0,++}) \\
F_3 &= \Re e F_{+-,++}; & F_5 &= \Re e F_{++,-} + \epsilon F_{00,-}; & F_7 &= F_{+-,-}; \\
F_9 &= F_{+-,-}; & F_{11} &= \Re e F_{+0,-}; & F_{13} &= \Re e F_{0+,-};
\end{aligned} \tag{27}$$

where the helicity-tensor elements depend on W^2, Q^2 and χ ; ϵ, ϵ' are again given by formula (16).

Azimuthal selection, applied in the same way as in III.C, makes it possible to derive from formula (26) six independent azimuthal correlations, from which eight independent structure functions can be extracted. Those correlations are the following :

(using the system of variables φ, φ_a) :

$$(2\pi)^{-1} \int F d\varphi = F_1 - 2\sqrt{\epsilon(1+\epsilon)}F_2 \cos \varphi_a - 2\epsilon F_3 \cos 2\varphi_a \tag{28}$$

(using the system of variables $\varphi, \varphi - \varphi_a$) :

$$(2\pi)^{-1} \int F d\varphi = F_1 - 2\epsilon' F_5 \cos 2(\varphi - \varphi_a) \tag{29}$$

(using the system of variables φ, φ_a) :

$$(2\pi)^{-1} \int F d\varphi_a = F_1 + \epsilon\epsilon' F_7 \cos 2\varphi \quad (30)$$

(using the system of variables $\varphi, 2\varphi_a - \varphi$) :

$$(2\pi)^{-1} \int F d\varphi = F_1 + \epsilon\epsilon' F_9 \cos(2\varphi_a - \varphi) \quad (31)$$

(using the system of variables $\varphi, 2\varphi - \varphi_a$) :

$$(2\pi)^{-1} \int F d\varphi = F_1 + 2\epsilon'\sqrt{\epsilon(1+\epsilon)}F_{11} \cos(2\varphi - \varphi_a) \quad (32)$$

(using the system of variables $\varphi, 2\varphi - 3\varphi_a$) :

$$(2\pi)^{-1} \int F d\varphi = F_1 + 2\epsilon'\sqrt{\epsilon(1+\epsilon)}F_{13} \cos(2\varphi - 3\varphi_a) \quad (33)$$

Here again we have assumed that there is no distortion of the φ -distribution due to the apparatus, since $\varphi \simeq \varphi^{\text{lab}}$ (formula (9)) and we may consider that both electrons are tagged in a cylindrically symmetric way.

On the other hand (as in configuration 2), given the limited acceptance of the central detector, ensuring the absence of distortion of the φ_a distribution is a critical problem that will be treated in the Appendix.

IV. APPLICATIONS OF CONFIGURATION 4

We shall now consider practical applications of the 8-term formula (26) computed in III.D. The processes considered are, here again, $\gamma\gamma$ production of muon and pion pairs. In that case one gets from formula (6), assuming $m_a^2, m_b^2 \ll W^2$:

$$d\text{Lips} = \frac{1}{(4\pi)^7} dx dx' dQ^2 dQ'^2 d\varphi^{\text{lab}} d\cos\chi d\varphi_a \quad (34)$$

Equating φ^{lab} with φ (see (9)), and taking account of (10) (with $Q' \rightarrow 0$), this expression transforms into :

$$d\text{Lips} = \frac{1}{(4\pi)^7} \frac{dx}{x} \frac{dW^2}{s} dQ^2 dQ'^2 d\varphi d\cos\chi d\varphi_a \quad (35)$$

Returning to formula (1), and using (16) and again (10), we obtain

$$\sigma = \frac{\alpha^4}{2\pi^3} \int W dW \frac{dQ}{Q} \frac{dQ'}{Q'} \frac{dx}{x} d\cos\chi \cdot \frac{1}{(W^2 + Q^2)^2} \left(1 - x + \frac{x^2}{2}\right) \left(1 - \frac{W^2 + Q^2}{xs} + \frac{(W^2 + Q^2)^2}{2x^2s^2}\right) F d\varphi d\varphi_a \quad (36)$$

For our computations we have fixed a number of limits on the various integration parameters chosen. Those limits are the following :

- (i) $Q' > 5 \times 10^{-3} E_0$ (corresponding approximately to a minimal tagging angle of 5 mrad for e'), and $Q' < W/20$ (in order to justify the neglect of φ -dependence of W in formula (10), and at the same time to motivate the neglect of longitudinal polarization of the photon γ').
- (ii) $Q < W$ (again because of formula (10)), and $Q > W/4$ (in order to ensure a significant contribution of the longitudinal polarization of γ).
- (iii) $W^2 > 4mE_0$ (see Appendix).
- (iv) Once again because of formula (10), we set : $x < 0.7$; $x' < 0.7$, or equivalently $x > (W^2 + Q^2)/(2.8E_0^2)$.
- (v) $x > Q/2E_0$ and $x < (W^2 + Q^2)/(4QE_0)$ (see Appendix) ; these limits, combined with those defined in (iv), also induce further limitations of Q and W .
- (vi) Finally, because of the particular sensitivity of polar-angle ranges close to 0 resp. π with regard to possible cuts located there, we set : $-0.9 < \cos\chi < 0.9$.

Using formula (36) and replacing $\int F d\varphi$, resp. $\int F d\varphi_a$, by their expressions given in formulas (28)-(33), and then integrating over the variables χ, x, Q', Q and W , we have obtained, for each particular process and model considered, the six angular distributions $d\sigma/d\Phi$, where Φ is to be identified respectively with $\varphi_a, \varphi - \varphi_a, \varphi, 2\varphi_a - \varphi, 2\varphi - \varphi_a, 2\varphi - 3\varphi_a$. In Figures 2-5 we are showing, for Φ ranging between 0 and π (notice that, in the Φ range between π and 2π , symmetric values are obtained), the normalized distributions $d\hat{\sigma}/d\Phi$, defined in such a way that $\int (d\hat{\sigma}/d\Phi) d\Phi = \pi$.

Let us remark that, since the structure functions F_2 , F_{11} and F_{13} are changing sign with $\cos \chi$, the distributions in $\varphi_a, 2\varphi - \varphi_a, 2\varphi - 3\varphi_a$ have been computed by integrating only over half of the $\cos \chi$ range, i.e. $0 < \cos \chi < 0.9$.

Fig. 2 shows the azimuthal correlations obtained for muon pair production, assuming a beam energy of 5 GeV. We have checked that the shapes of those curves remain practically unchanged when one goes over to a much higher beam energy ($E_0 = 100$ GeV).

Figs. 3 and 4 show, again for $E_0 = 5$ GeV, the corresponding correlations obtained for $\pi^+\pi^-$ production, as computed respectively in two different models : the perturbative-QCD model proposed by Brodsky and Lepage [18], as extended by Gunion et al. [19] for $\gamma\gamma^*$ collisions, with the pion wave function given by Chernyak and Zhitnitsky [20] ; and the finite-size model of Poppe [21], where the amplitude of the Born-term calculation is multiplied by an overall form factor. Notice that here the lower limit introduced for W is $4m_\pi E_0 \simeq 1.7$ GeV.

Fig. 5 shows the analogous curves obtained for $\pi^0\pi^0$ production at the same beam energy, using again the Brodsky-Lepage model with the Chernyak-Zhitnitsky wave function.

The values of integrated cross sections (obtained by replacing F by F_1 in (36)) are given in Table 1, where they are compared with those of the “theoretical background” (see Ref. [22]), i.e. of the contribution of the Feynman diagram shown in Fig. 6, computed within the same kinematic limits. For $\pi^+\pi^-$ production, we here use a simple VDM model involving only ρ -exchange. It is seen that this background remains insignificant as compared to the signal (in the case of $\pi^0\pi^0$ production, it is of course strictly zero). As regards the interference term between the diagrams of Figs. 1 and 6, it is reduced to zero if, instead of identifying particle a with the muon (resp. pion) of either positive or negative charge, we average over those two options.

Notice that in the figures of Table 1 we have included a factor of 2 in order to account for the possibility of performing a symmetric measurement (e tagged at large angle, e' at small angle ; and conversely).

Finally it is to be emphasized that, if one uses a Monte-Carlo program taking account of

all experimental acceptances, one would certainly be allowed to suppress some of the severe kinematic restrictions here introduced.

V. DISCUSSION AND CONCLUSION

Azimuthal correlations are, to a large extent, a new approach to the study of photon-photon collisions. Till now practically all experimental efforts in two-photon physics have been concentrated on measuring $\gamma\gamma$ cross sections or the “photon structure function” F_2^γ (i.e. the $\gamma\gamma^*$ total hadronic cross section), which correspond to the diagonal structure function F_1 in the formalism presented here. Azimuthal correlations should allow one to determine, depending on the configuration considered, one, two, three or seven additional independent structure functions. It should be emphasized that in principle F_1 does not contain a larger amount of physical information than the others. Its special status arises only from the fact that it is more easily determined ; in addition, due to the Schwarz inequality, it is larger than (or at least equal to) any of the interference structure functions. In that sense it appears as the *primus inter pares*, but not more. In other words : azimuthal correlations should allow one to multiply the physical information, obtained in a number of two-photon processes, by a factor of 2, 3, 4 or 8.

As can be seen for instance by comparing Figs. 3 and 4, two different dynamic models for a given process can lead to very different shapes of the azimuthal-correlation curves, implying that the values of the structure functions involved are very different as well. Thus azimuthal correlations should provide one with a powerful tool for checking dynamic models. Actually we may safely state that no model will survive this kind of check if it is not entirely realistic. There is however a price to pay for this achievement : there are more or less stringent experimental constraints that should be satisfied. In configurations 1 and 3, what is required is the possibility of tagging the outgoing electrons (and of measuring their azimuthal angles) at small scattering angles. In configuration 2, an “almost 4π ” central detector appears necessary. In configuration 4 both requirements are combined, and in addition, since the

measured cross section is sharply reduced by significant cuts in most of the parameters to be measured, a very high machine luminosity is required.

However, as already noticed above, the use of a Monte-Carlo program should allow one to relax to some extent the constraints here defined and thus to increase the integrated cross section.

In any case, and whatever the particular $\gamma\gamma$ process and the configuration considered, azimuthal-correlation measurements in muon pair production under the same conditions should always be used as a test, so as to check the validity of the approximations applied. Notice that in general it would be more problematic to use electron pairs for that purpose, given the complications due to exchange between scattered and produced electrons.

Let us finally remark that, for checking perturbative QCD, it appears particularly interesting to look for azimuthal correlations in the two-photon production of quark pairs (notice that, in the quark-parton model, the azimuthal-correlation curves predicted for the production of light-quark pairs are the same as for $\mu^+\mu^-$ production). Measuring azimuthal angles of jets is probably a difficult, but not impossible, task for experimentalists. Another option would be to investigate azimuthal correlations in the inclusive production of one hadron (plus anything).

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APPENDIX: EXPERIMENTAL CONSTRAINTS TO BE APPLIED TO CONFIGURATIONS 2 AND 4

Our problem is how to minimize azimuth-dependent cuts in $\cos \chi$, induced by the limited acceptance of the central detector. For simplicity, we shall stick to the case of production of particle-antiparticle pairs ($b = \bar{a}$). Then we can use formulas (A10) - (A12) of the Appendix of Ref. [14]. We shall however rewrite those formulas, using slightly different notations (and in addition, as regards formula (A12), exchanging the variables pertaining to the left- and right-hand vertex of Fig. 1). We thus get

$$\beta \Lambda^{1/2}(W^2, -Q^2, -Q'^2) \cos \chi = Q'^2 - W^2 - Q^2 + 4E_a \{ E_0(1 + \beta_a \cos \psi) - E' [1 + \beta_a \cos \theta' \cos \psi - \beta_a \sin \theta' \sin \psi \cos(\varphi^{\text{lab}} - \varphi_a^{\text{lab}})] \} \quad (\text{A1})$$

with E_a , the lab energy of particle a , given by

$$E_a = \frac{E_X W^2 \pm \tilde{p}_X \left[W^4 - 4m_a^2(E_X^2 - \tilde{p}_X^2) \right]^{1/2}}{2(E_X^2 - \tilde{p}_X^2)} \quad (\text{A2})$$

where we have defined

$$E_X = 2E_0 - E - E' \quad (\text{A3})$$

$$\begin{aligned} \tilde{p}_X &= -E(\cos \theta \cos \psi + \sin \theta \sin \psi \cos \varphi_a^{\text{lab}}) + \\ &E'[\cos \theta' \cos \psi - \sin \theta' \sin \psi \cos(\varphi^{\text{lab}} - \varphi_a^{\text{lab}})] \end{aligned} \quad (\text{A4})$$

and where in addition we have used the following definitions : θ, θ' are the lab scattering angles of e, e' , while ψ is the lab polar emission angle of particle a with respect to e ; $\beta = (1 - 4m_a^2/W^2)$; $\beta_a = (1 - m_a^2/E_a^2)^{1/2}$. Notice that in formula (A2) only solution $+$ is valid when $W^2 > 2m_a E_X$.

It is to be remarked that $\cos \chi$ is implicitly depending on φ, φ_a through its explicit dependence on φ^{lab} and φ_a^{lab} (see formulas (A9), (A13) of Ref. [14], showing the relations between azimuthal angles in the lab and the c. m. frame).

In the configuration (2 or 4) considered here, we let Q' (and correspondingly θ') go to zero, so that formula (A1) becomes

$$1 + \cos \chi \simeq \frac{4E_a x' E_0 (1 + \beta_a \cos \psi)}{W^2 + Q^2} \simeq \frac{E_a (1 + \beta_a \cos \psi)}{x E_0} \quad (\text{A5})$$

On the other hand we assume $W^2 > 4m_a E_0$, which entails (according to (A3), setting $x, x' > 0.7$) that $W^4 > 8m_a^2 E_X^2$. Therefore only solution + is to be considered in (A2) ; in addition it becomes possible to make a convergent series expansion, in m_a^2/W^2 , of the radical in (A2). That formula thus becomes

$$E_a \simeq \frac{W^2}{2(E_X - \tilde{p}_X)} - \tilde{p}_X \frac{m_a^2}{W^2} + O\left(\frac{m_a^4}{W^4}\right) \quad (\text{A6})$$

where we shall retain only the first term on the right-hand side. Substituting that expression of E_a into (A5), we get, taking account of (A3), (A4) with $\theta' \rightarrow 0$:

$$1 + \cos \chi = \frac{(1 + \beta_a \cos \psi) W^2}{2x E_0 [2E_0 - E(1 - \cos \theta \cos \psi + \sin \theta \sin \psi \cos \varphi_a^{\text{lab}}) - E'(1 + \cos \psi)]} \quad (\text{A7})$$

We now assume that the acceptance of the central detector is given by : $\psi_0 < \psi < \pi - \psi_0$, with $\psi_0 \ll 1$ rad. We then compute the cuts in $\cos \chi$ induced by the forward and backward cut in ψ , neglecting terms in ψ_0^2 .

(i) Forward cut :

$$\begin{aligned} |\Delta(\cos \chi)| &= \left| (1 + \cos \chi)_{\psi=\psi_0} - (1 + \cos \chi)_{\psi=0} \right| \\ &\simeq (1 + \beta_a) \left| \frac{W^2}{2x E_0 (2x' E_0 - Q^2/2E_0 + \psi_0 E \sin \theta \cos \varphi_a^{\text{lab}})} - \frac{W^2}{2x E_0 (2x' E_0 - Q^2/2E_0)} \right| \\ &\simeq (1 + \beta_a) \frac{W^2}{W^2 + (1-x)Q^2} \frac{\psi_0 E \sin \theta |\cos \varphi_a^{\text{lab}}|}{2x' E_0 (1 - Q^2/s)} < \frac{Q \psi_0}{x' E_0 (1 - Q^2/s)} \end{aligned} \quad (\text{A8})$$

Setting $x' > Q/[E_0(1 - Q^2/s)]$, i.e. $x < (W^2 + Q^2)(1 - Q^2/s)/(4QE_0)$, one is led to $|\Delta \cos \chi| < \psi_0$. For simplicity (and since values of Q^2/s much smaller than 1 are largely predominating), we shall approximate the above-defined upper limit of x by requiring only $x < (W^2 + Q^2)/(4QE_0)$.

(ii) Backward cut :

$$\begin{aligned}
|\Delta(\cos \chi)| &= \left| (1 + \cos \chi)_{\psi=\pi-\psi_0} - (1 + \cos \chi)_{\psi=\pi} \right| \\
&\simeq (1 - \beta_a) \left| \frac{W^2}{2xE_0(2xE_0 + \psi_0 E \sin \theta \cos \varphi_a^{\text{lab}})} - \frac{W^2}{4x^2 E_0^2} \right| \\
&\simeq (1 - \beta_a) \frac{W^2}{4x^2 E_0^2} \frac{\psi_0 E \sin \theta |\cos \varphi_a^{\text{lab}}|}{2xE_0} < \frac{W^2 Q \psi_0}{8x^3 E_0^3} \tag{A9}
\end{aligned}$$

Setting $x > \sup [W/(2E_0), Q/(2E_0)]$, we are led here again to $|\Delta \cos \chi| < \psi_0$. Notice that in configuration 4, having set $Q \lesssim W$, that condition simply becomes $x > W/(2E_0)$.

It is easily checked that the lower and upper limit thus fixed for x are compatible without imposing any further constraint on W and Q .

With $\psi_0 = 0.1$ rad, the azimuth-dependent cut in $\cos \chi$, induced either by the forward or the backward cut in x , is reduced to less than 5 % of phase space.

We have verified a posteriori that, with the constraints here defined, terms that we have neglected in the expressions of $\cos \chi$ and E_a have only negligible effects on $\Delta \cos \chi$.

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FIGURES

FIG. 1. Feynman diagram for the reaction $e e' \rightarrow e e' a b$ involving the exchange of two spacelike photons.

FIG. 2. Azimuthal correlations ($d\hat{\sigma}/d\Phi$) computed in configuration 4, under the conditions defined in the text, for the reaction $e e' \rightarrow e e' \mu^+ \mu^-$ at a beam energy of 5 GeV. Solid line : $\Phi = \varphi_a$; dotted line : $\Phi = \varphi - \varphi_a$; long-dashed line : $\Phi = \varphi$; short-dashed line : $\Phi = 2\varphi_a - \varphi$; long-dashed/dotted line : $\Phi = 2\varphi - \varphi_a$; short-dashed/dotted line : $\Phi = 2\varphi - 3\varphi_a$.

FIG. 3. Same as Fig. 2, for the reaction $e e' \rightarrow e e' \pi^+ \pi^-$ computed in the Brodsky-Lepage model [18], as extended by Gunion et al. [19], using the Chernyak-Zhitnitsky wave function [20].

FIG. 4. Same as Fig. 2, for the reaction $e e' \rightarrow e e' \pi^+ \pi^-$ computed in the finite-size model of Poppe [21].

FIG. 5. Same as Fig. 2, for the reaction $e e' \rightarrow e e' \pi^0 \pi^0$ computed in the Brodsky-Lepage model [18], as extended by Gunion et al. [19], using the Chernyak Zhitnitsky wave function [20].

FIG. 6. Feynman diagram for the reaction $e e' \rightarrow e e' a b$ involving the exchange of one spacelike and one timelike photon.

TABLES

Table 1. Comparison between the integrated cross sections, computed in configuration 4 under the conditions defined in the text, of the reactions $e e' \rightarrow e e' a b$ as described by the Feynman diagrams of Fig. 1 (signal) and of Fig. 6 (background) respectively.

$a b$	E_0 (GeV)	σ_{signal} (10^{-40}cm^2)	$\sigma_{\text{background}}$ (10^{-40}cm^2)
$\mu^+ \mu^-$	5	24 020	88.6
$\pi^+ \pi^-$	5	^a 142	0.215
$\pi^+ \pi^-$	5	^b 249	0.215
$\pi^0 \pi^0$	5	20	—
$\mu^+ \mu^-$	100	222	0.283

^aBrodsky-Lepage [18]

^bPoppe [21]

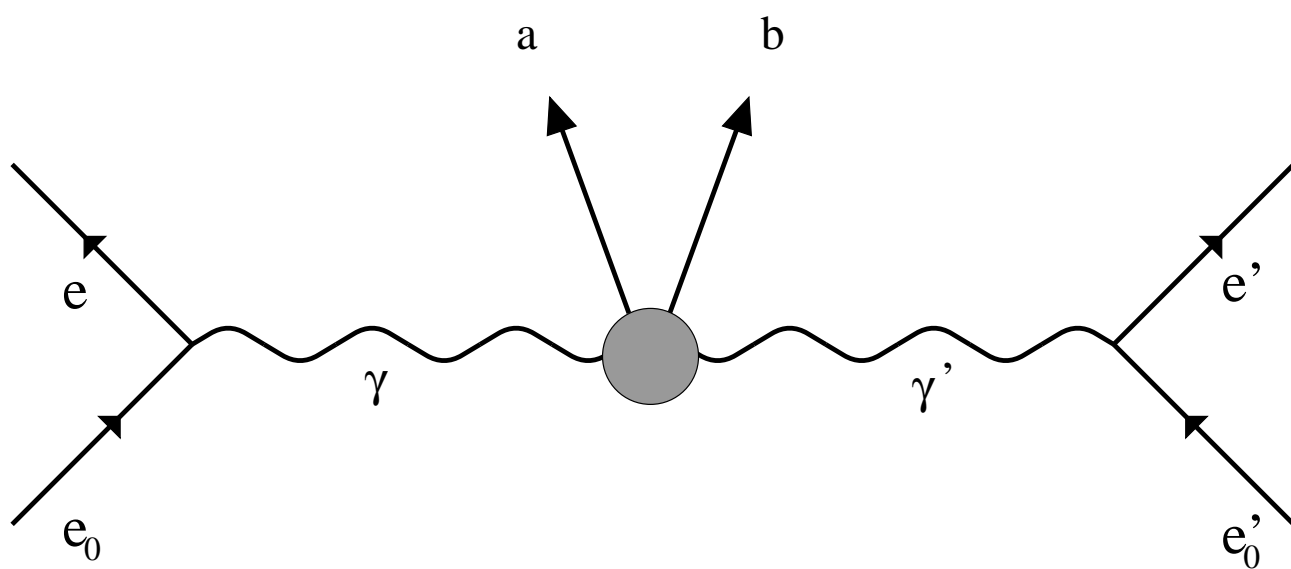


Fig. 1

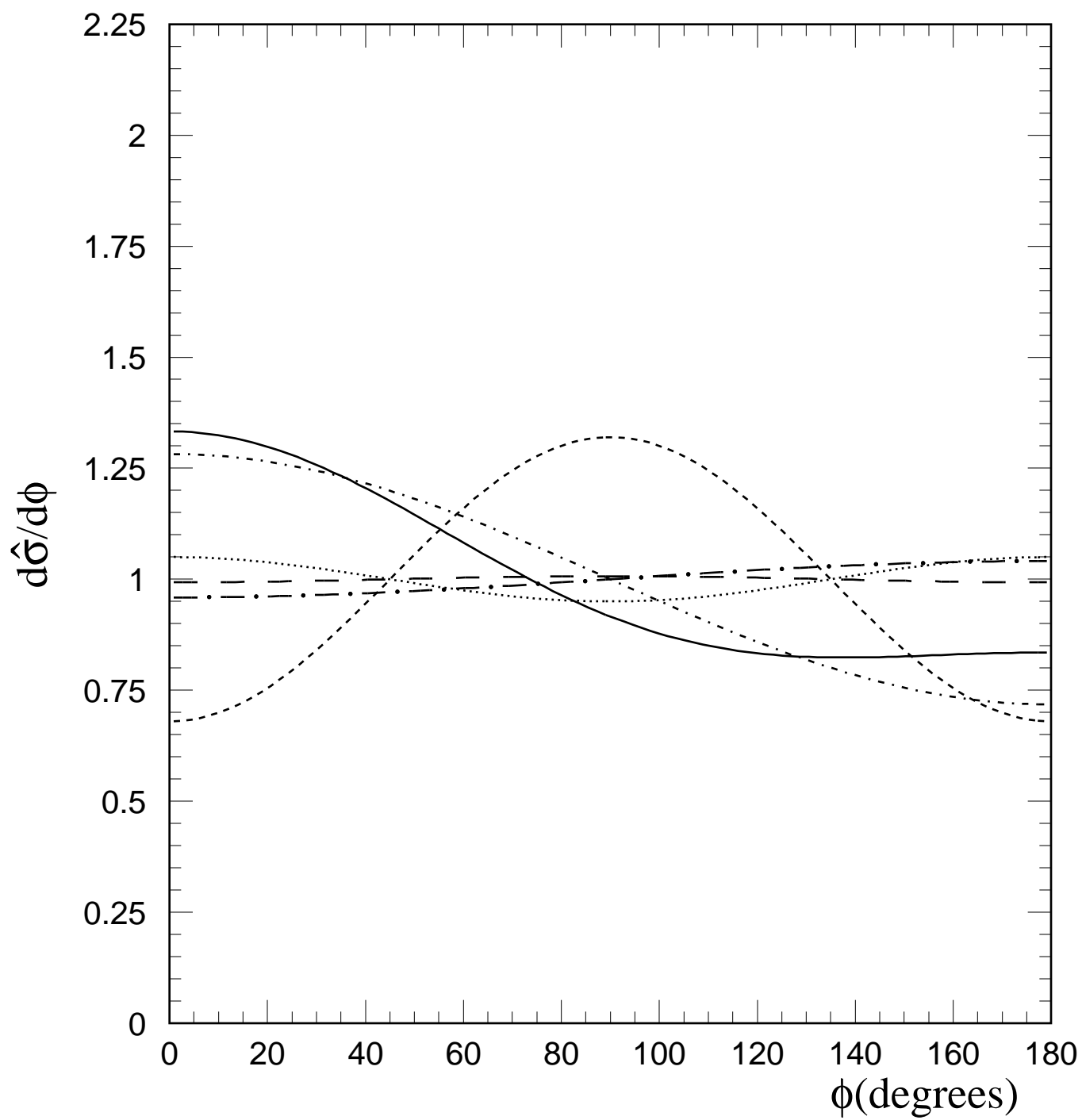


Fig. 2

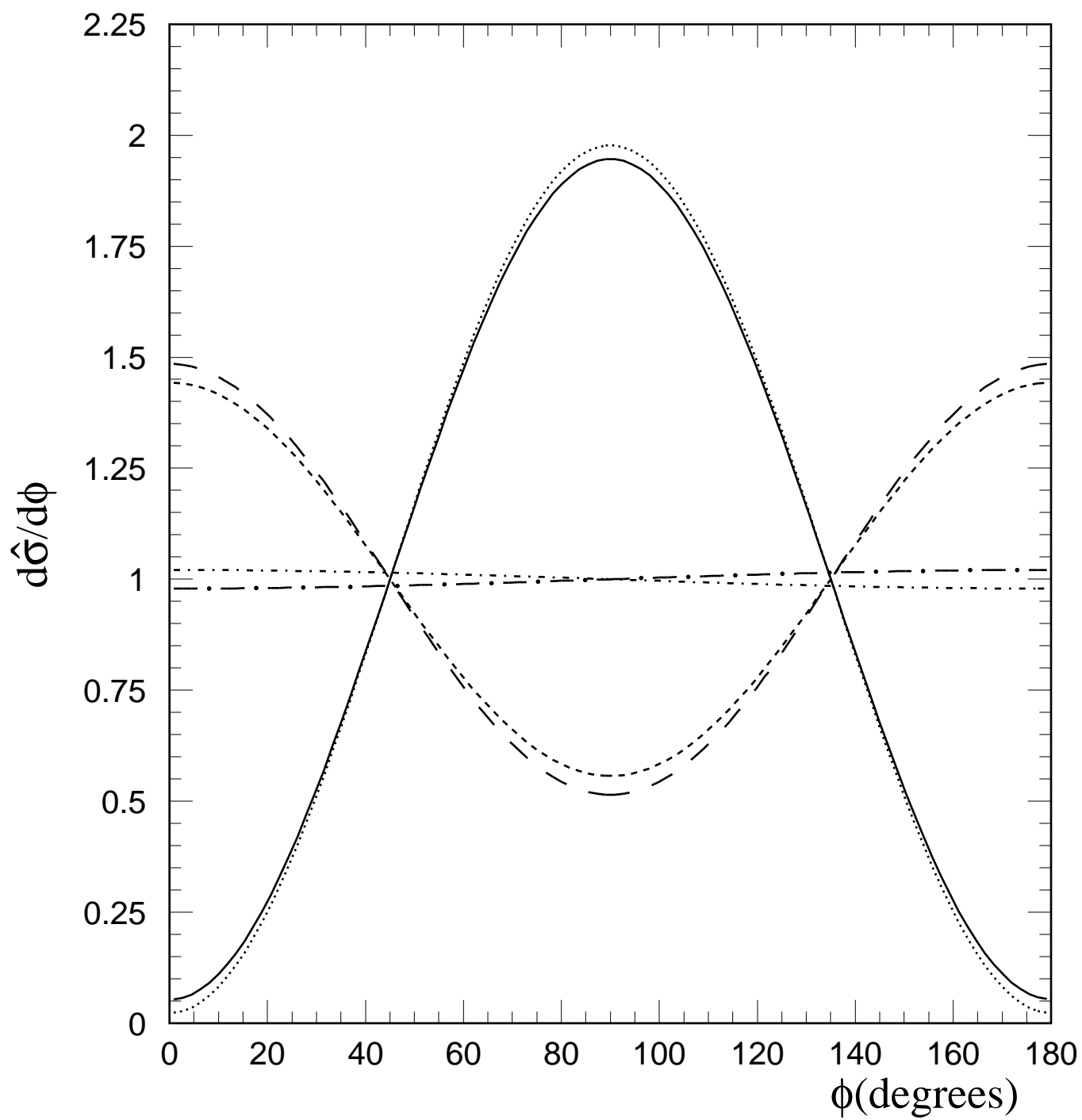


Fig. 3

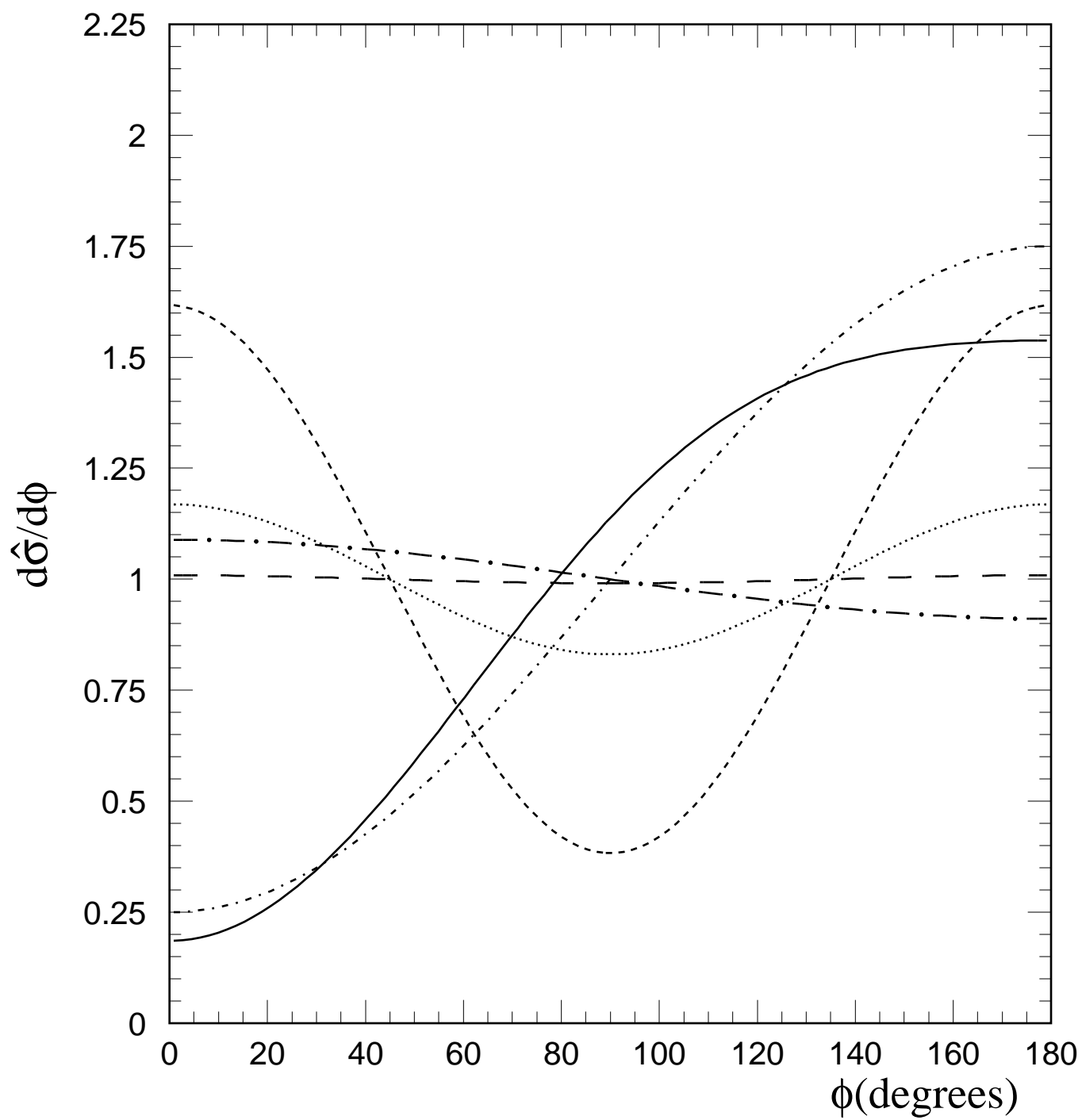


Fig. 4

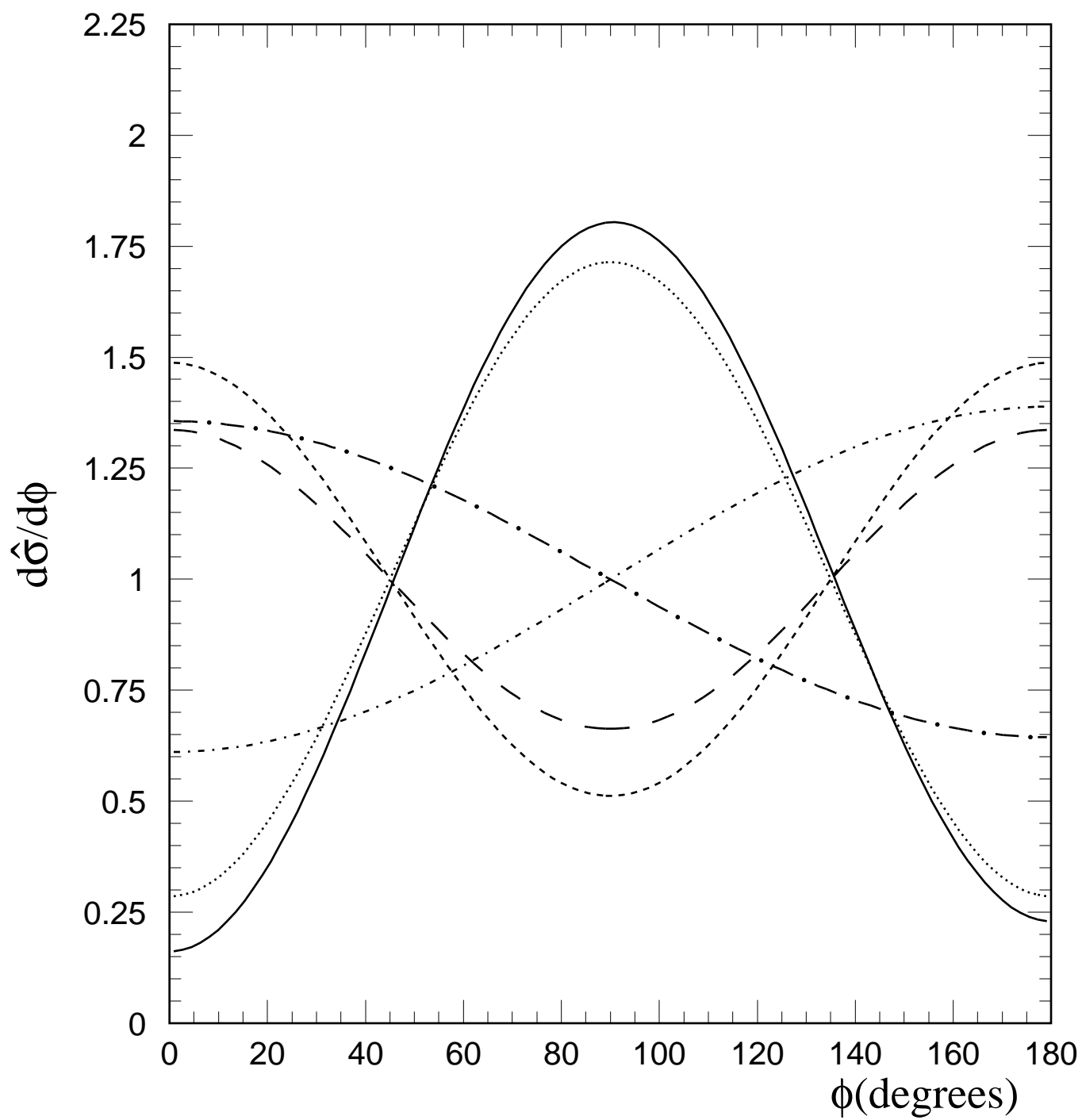


Fig. 5

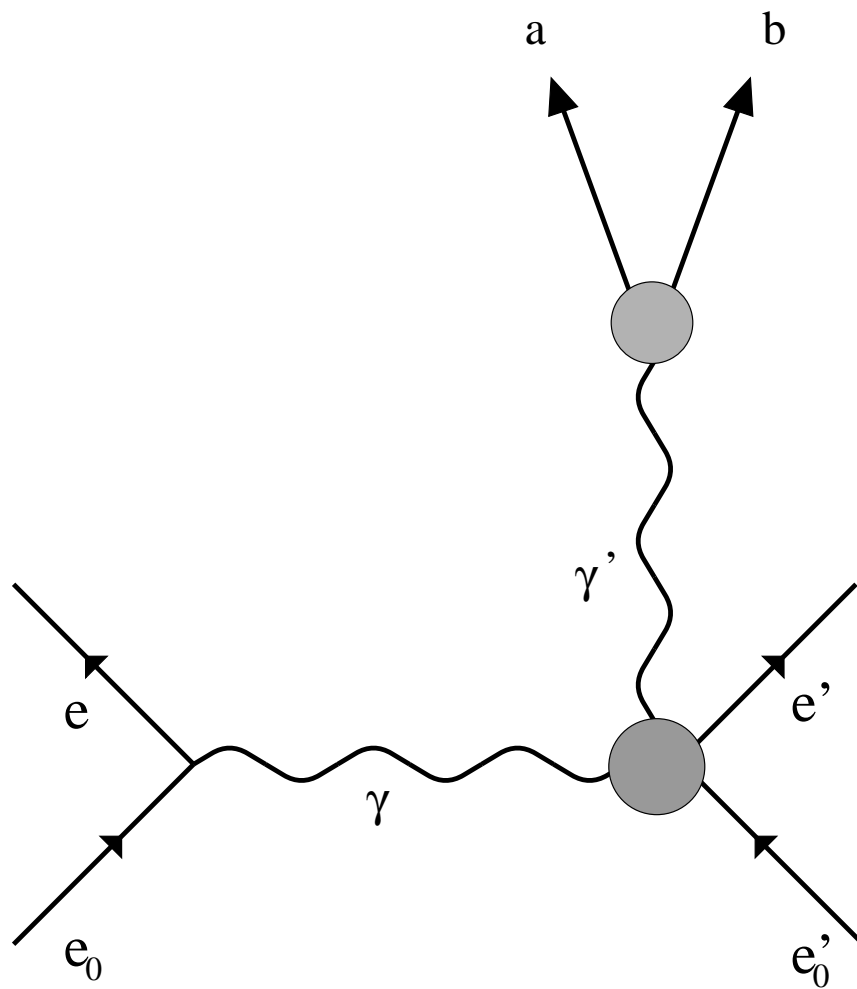


Fig. 6